Limiti notevoli

$$\lim_{x\to+\infty} x^\alpha = +\infty\,, \qquad \lim_{x\to 0^+} x^\alpha = 0\,, \qquad \alpha > 0$$

$$\lim_{x\to+\infty} x^\alpha = 0\,, \qquad \lim_{x\to 0^+} x^\alpha = +\infty\,, \qquad \alpha < 0$$

$$\lim_{x\to\pm\infty} \frac{a_n x^n + \ldots + a_1 x + a_0}{b_m x^m + \ldots + b_1 x + b_0} = \frac{a_n}{b_m} \lim_{x\to\pm\infty} x^{n-m}$$

$$\lim_{x\to+\infty} a^x = +\infty\,, \qquad \lim_{x\to-\infty} a^x = 0\,, \qquad a > 1$$

$$\lim_{x\to+\infty} a^x = 0\,, \qquad \lim_{x\to-\infty} a^x = +\infty\,, \qquad a < 1$$

$$\lim_{x\to+\infty} \log_a x = +\infty\,, \qquad \lim_{x\to 0^+} \log_a x = -\infty\,, \qquad a > 1$$

$$\lim_{x\to+\infty} \log_a x = -\infty\,, \qquad \lim_{x\to 0^+} \log_a x = +\infty\,, \qquad a < 1$$

$$\lim_{x\to\pm\infty} \sin x\,, \qquad \lim_{x\to\pm\infty} \cos x\,, \qquad \lim_{x\to\pm\infty} \tan x \qquad \text{non esistono}$$

$$\lim_{x\to\left(\frac{\pi}{2}+k\pi\right)^\pm} \tan x = \mp\infty\,,\ \forall k \in \mathbb{Z}\,, \qquad \lim_{x\to\pm\infty} \arctan x = \pm\frac{\pi}{2}$$

$$\lim_{x\to\pm 1} \arcsin x = \pm\frac{\pi}{2} = \arcsin(\pm 1)$$

$$\lim_{x\to+1} \arccos x = 0 = \arccos 1\,, \qquad \lim_{x\to-1} \arccos x = \pi = \arccos(-1)$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1\,, \qquad \lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x\to\pm\infty} \left(1+\frac{a}{x}\right)^x = \mathrm{e}^a\,, \quad a \in \mathbb{R}\,, \qquad \lim_{x\to 0} (1+x)^{\frac{1}{x}} = \mathrm{e}$$

$$\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log a}\,,\ a > 0; \quad \text{in particolare,} \quad \lim_{x\to 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \log a\,, \quad a > 0; \quad \text{in particolare,} \quad \lim_{x\to 0} \frac{\mathrm{e}^x - 1}{x} = 1$$

$$\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha\,, \quad \alpha \in \mathbb{R}$$

Fonte: "Analisi Matematica 1", C.Canuto A.Tabacco